1

ELECTROSTATICS

CONTENTS

Introduction, Electric field lines, electric flux and Gauss law, the divergence of E, Curl of E, Application of Gauss's law.

i) Electric field due to uniform charged sphere ii) Electric field due to charged cylinder Gaussian Pillbox, Poisson's equation, Laplace's equation, Uniqueness theorem (First and second)

1.1 Introduction :

It is our common experience that, when a hard rubber comb is rubbed on a piece of wool, it acquires the ability to attract small pieces of paper. similar phenomenon occurs if we rub a glass rod with a piece of silk cloth, why does this happen? The answer to this question comes from a branch of physics known as ***electrostatics.***

Electrostatics is the physics of stationary electric charges. The hard rubber comb or glass rod become electrically charged as a result of rubbing. In fact, electrons are transferred from the wool to the rubber so that the wool becomes positively charged & the rubber comb, negatively charged. There are only two kinds of electric charges; positive charges & negative charges. like mass, electric charge is an intrinsic property of matter. Electric charge is quantized. the quantum of charge being e. The charge on any body will be integral multiple of e. ie. $q = \pm ne$

where n = 1, 2, 3, 4 ....

The magnitude of e is equal to 1.6021×10^{-19} coulomb and is called the electronic charge. Charge is a scalar quantity. During any process the net electric charge of an isolated system remains constant or we can say that charge is conserved. Here we mean an isolated system as a system through the boundary of which no charge is allowed to escape or enter.

We start our study with coulomb's law for electrostatics forces because it is fundamental. We can deduce from it the basic concepts and laws of electrostatics.

1.2 Electric Field Lines and Electric Flux :

The Electric field produced by a point charge is given by $E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}\vec{r}$ obeys the inverse square law. If the distance r is doubled, the strength of the field falls by a factor of four, and so on. Moreover, the field is spherically symmetric because it depends only on r, the spherical polar co-ordinate that stands for the distance from the source 2 to the field point. Michael Faraday conceived the idea of picturizing the electric field by lines of force.

Following are the properties of lines of force representing the electric field E :

a) Lines of force starts from positive charge and terminates on netative charge.

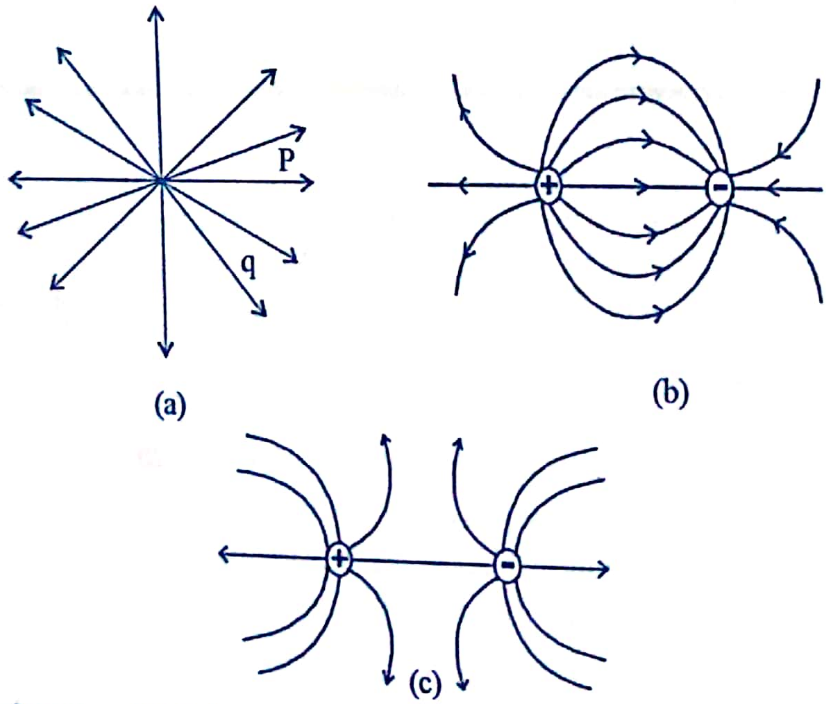
b) There is tension in every line; it tries to pull its ends together.

c) There is repulsion between the lines. The rules for getting the Electric Field at any point of the line - map of the field are as follows :

d) The field direction at any point is along the tangent to the line of force passing through that point and along the direction of line of force.

e) The field intensity is proportional to the number of lines passing through unit area held normal to the local field direction.

Figure shows the lines of force for some simple arrangements of point charge.



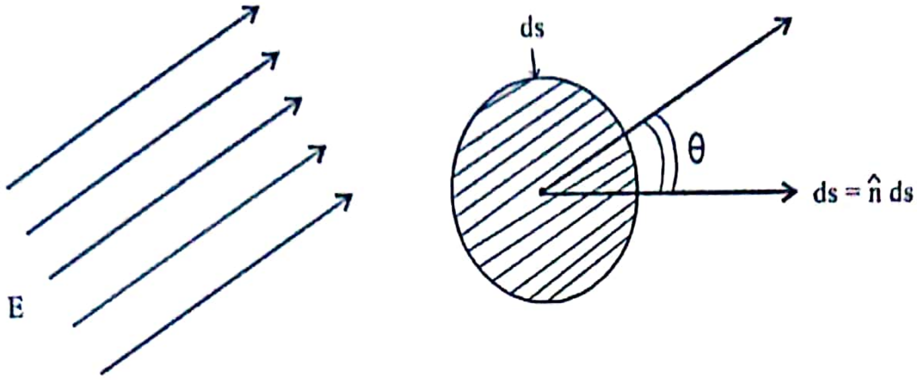
[Lines of force of (a) A positive pt. charge.
(b) Two equal & opposite point charges (electric dipole) (c) Two equal positive charges.]

Lines of force form a simple construction that enable us to visualise the electric field.

The unit of electric field is volts/meter. If we draw 9×10^9 lines from 1 coulomb of positive charge, then a field strength of N/C will correspond to $\frac{1}{4\pi}$ lines per meter. Lines of force are analoges to the flow lines of the flow of a liquid. No two lines of force can interest. Also no lines of force of electric field E can form a closed loop.

1.3 Electric Flux And Gauss Law :

Flux of electric field



[Fig. Electric Flux through an area element]

figure shows a small area ds in a uniform electric field $\vec{E}$. The area can be represented by a vector $\vec{ds}$ whose magnitude is proportional to the measure of area and whose direction is normal to the area consider scalar product $\vec{E}.\vec{ds} = (E.\hat{n}\,ds)$ then $d\phi = E\cos\theta ds$ is called flux $\vec{E}$ of through the area ds or it gives the flux as the component of $\vec{E}$ along $\vec{ds}$ (i.e. along the normal to area) multiplied by area.

The two properties of flux are :

a) $d\phi$ is a scalor

b) $d\phi$ is ≥ 0 depending on whether

$\theta \leq \frac{\pi}{2}$ resp

$\therefore$ flux of $\vec{E}$

The flux of $\vec{E}$ over an arbitrary surface S is given by the integral.

$$\phi = \int d\phi = \int_s \vec{E}.\vec{d}s = \int \vec{E}.\hat{n}\,ds \qquad \text{----- (1)}$$

In case the suraces happens to be closed one, we write equation (1) as

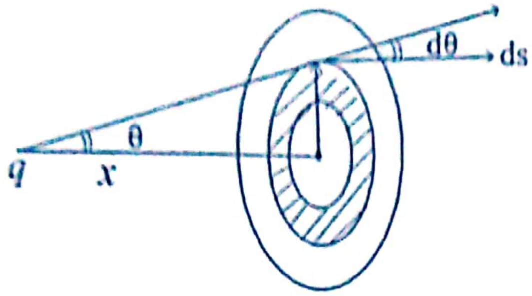
$$\phi = \oint \vec{E}.\hat{n}\,ds \qquad \text{----- (2)}$$

When ds is drawn along the outward normal $\hat{n}$ out, the flux integral in equation (2) is known as total outward flux of $\vec{E}$ for the given closed surface. Clearly the total inward flux is the negative of the total outward flux for a closed surface.

The unit of ϕ is Vm or Nm^2/C

Example :

A circular area of radius a has a pt. charge q on its axis at a dist. x from it. Calculate the flux of electric field due to q through the circular area.



We consider area element ds of radius r & width dr of circle. Taking the direction of vector ds away from q, the flux through the element is

$$d\phi = \vec{E}.\hat{n}ds = \frac{q(\cos\theta)2\pi rdr}{4\pi\in_0\left(x^2+r^2\right)}$$

Using $\cos\theta = \frac{x}{\sqrt{x^2+r^2}}$ etc. we find total flux ϕ $\xi = x^2 + a^2$

$$\phi = \frac{qx}{4\in_0}\int_{r=0}^{a}\frac{2r\,dr}{\left(x^2+r^2\right)^{3/2}} = \frac{qx}{4\in_0}\int_{\xi=x^2}^{-3/2}\xi d\xi$$

$$= \frac{q}{2\in_0}\left(1-\frac{x}{\sqrt{x^2+\alpha^2}}\right)$$

$$= \frac{q}{2\in_0}(1-\cos\alpha) \text{ where } x = \tan^{-1}\left(\frac{\alpha}{a}\right)$$

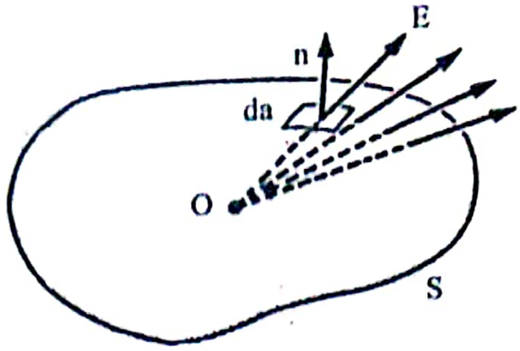
1.4 Gauss's Law :

An important relationship exists between the integral of the normal component of the electric field over a closed surface and total charge enclosed by the surface. This relationship, known as Gauss's law.

$$\oint_s E.n\,da = \frac{1}{\in_0}\sum q_i$$

The electric field at point r due to a point charge q located at the origin is

$$E(r) = \frac{q}{4\pi\in_0}\frac{r}{r^3}$$



[Fig. : An imaginary closed surfaces *S* that encloses a point charge at the origin.]

Consider the surface integral of the normal component of this electric field over a closed surface that encloses the origin and, consequently, the charge q ; this integral is just.

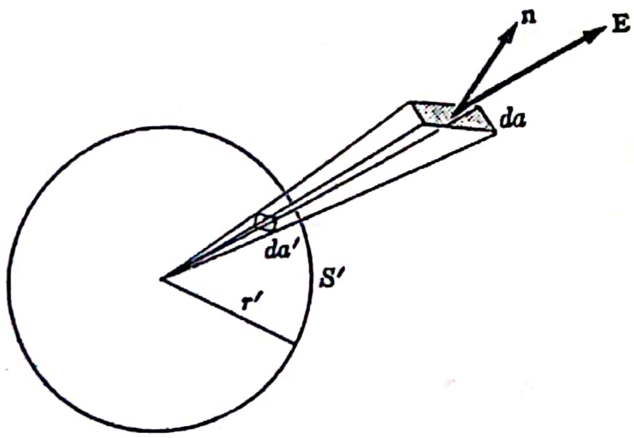
$$\oint_S E.n\, da = \frac{q}{4\pi \in_0} \oint_{S'} \frac{r.n}{r'^3} da$$

The quantity $(r/r).n\ da$ is the projection of da on a plane perpendicular to r. This projected area divided by r^2 is the solid angle subtended by da, which is written $d\Omega$. It is clear from Fig. that the solid angle subtended by da is the same as the solid angle subtended by da', an element of the surface area of the sphere S' whose center is at the origin and whose radius is r', It is then possible to write

$$\oint_S \frac{r.n}{r^3} da = \oint_{S'} \frac{r'.n}{r'^3} da' = 4\pi$$

which shows that

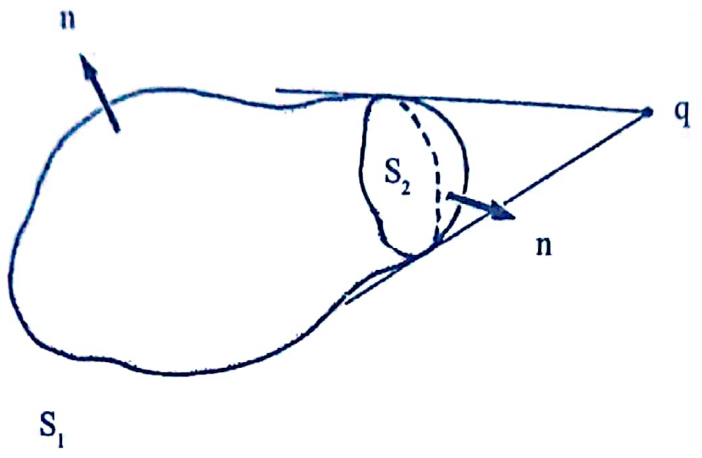
$$\oint_S E.n\, da = \frac{q}{4\pi \in_0} 4\pi = \frac{q}{\in_0}$$



[Fig : Construction of the spherical surface *S'* as an aid to evaluation of the solid angle subtended by *da*.]

If q lies outside of S, it is clear from Fig. that S can be divided into two areas S_1 and

S_2 each of which subtends the same solid angle at the charge q. For S_2, however, the direction of the normal is toward q, while for S_1 it is away from q. Therefore the contributions of S_1 and S_2 to the surface integral are equal and opposite, and the total integral vanishes. Thus if the surface surrounds a point charge q, the surface integral of the normal component



[Fig.: The closed surface S may be divided into two surfaces, S_1 and S_2 each of which subtend the same solid angle at q.]

of the electric field is q / ϵ_0, while if q lies outside the surface the surface integral is zero. The preceding statement applies to any closed surface, even to so-called re-entrant ones. A study of Fig. is sufficient to verify that this is indeed the case.

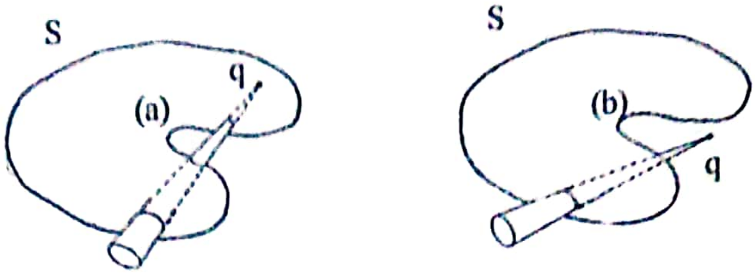
If several point charges $q_1, q_2, ...q_N$ are enclosed by the surface S, then the total electric field is given by the first term . Each charge subtends a full solid angle (4π); hence Eq. becomes

$$\oint_S E.n\,da = \frac{1}{\epsilon_0}\sum_{i=1}^{N} q_i \qquad \text{----- (a)}$$

This result can be readily generalized to the case of a continuous distribution of charge characterized by a charge density. If each element of charge $\rho\,dv$ is considered as a point charge, it contributes $\rho\,dv/\epsilon_0$ to the surface integral of the normal component of the electric field provided it is inside the surface over which we integrate. The total surface integral is then the sum of all contributions of this form due to the charge inside the surface. Thus if S is a closed surface which bounds the volume V,

$$\oint_S E.n\,da = \frac{1}{\epsilon_0}\int_V \rho\,dv \qquad \text{------ (b)}$$

Equation (a) and (b) are known as Gauss's law. The term on the left, the integral of the normal component of the electric field over the surface S, is sometimes called the flux of the electric field through S.



[**Fig. : An element of solid angle cutting the surface S more than once.**]

Gauss's law may be express in yet another form by using the divergence theorem. The divergence theorem states that

$$\int_S E.n\,da = \int_V \nabla.F\,dv$$

If this theorem is applied to the surface integreal of the normal component of E, it yields.

$$\oint_S E.n\,da = \int_V \nabla.E\,dv \qquad \text{------ (c)}$$

which, when substituted into Eq. (b), gives

$$\int_V \nabla.E\,dv = \frac{1}{\epsilon_0}\int_V \rho\,dv \qquad \text{------ (d)}$$

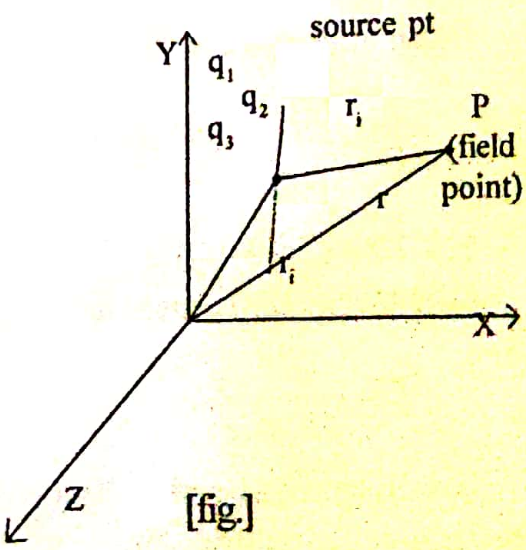
Equation (d) must be valid for all volumes, that is, for any choice of the volume V. The only way in which this can be true is if the integrands appearing on the left and on the right in the equation are equal. Thus the validity of Eq. (d) for any choice of V implies that

$$\nabla.E = \frac{1}{\epsilon_0}\rho \qquad \text{----- (e)}$$

This result may be thought of as a differential form of Gauss's law.

1.5 The Divergence of E: If we have several point charges. $q_1, q_2, q_3 \ldots\ldots q_n$ at distance $r_1, r_2, r_3 \ldots\ldots r_n$ from charge Q, then electric field of the source charge is

$$E(r) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i \qquad \text{----- (1)}$$



[fig.]

It is a function of position (r) since separation vector $\hat{r}_i$ depends on location of field point.

But if charge distribution is continuous over some region, then sum becomes integral

$$\therefore E(r) = \frac{1}{4\pi \in_0} \int \frac{1}{r^2} . \hat{r}\, dq$$

& if charge fills a volume as shown in figure, with charge per unit volume ρ then $dq = \rho\, d\tau'$ where $d\tau'$ is an element of volume, so

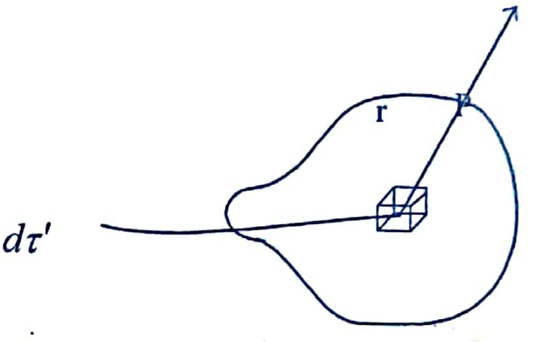
$$E(r) = \frac{1}{4\pi \in_0} \int_{all\ space} \frac{\rho(r')}{r^2} \hat{r}\, d\tau' \qquad \text{----- (2)}$$

Now taking divergence of E (i.e. $\nabla . E$)

We have,

$$\nabla . E = \frac{1}{4\pi \in_0} \int \nabla . \left(\frac{\hat{r}}{r^2} \right) \rho(r')\, d\tau'$$

where $r = r - r'$ ------ (3)



[Fig. Volume Charge ρ]

Now $\nabla . \left(\frac{\hat{r}}{r^2} \right) = 4\pi\, \delta^3 (r)$

Thus

$$\nabla . E = \frac{1}{4\pi \in_0} \int 4\pi\, \delta^3 (r - r')\, \rho(r')\, d\tau'$$

$$\therefore \nabla . E = \frac{1}{\in_0} \rho(r) \qquad \text{----- (4)}$$

This is Gauss's law in differential form i.e. $\nabla . E = \frac{\rho}{\in_0}$

integrating equation (4) over volume V and applying divergence theorem

$$\int_V \nabla . E\, d\tau = \oint_S E . da = \frac{1}{\in_0} \int_V \rho d\tau = \frac{1}{\in_0} Q_{enc}$$

1.6 The Curl of E :

Consider a point charge at the origin in this case.

$$E = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} \qquad \text{---- (1)}$$

Now to calculate curl of E we calculate the line. integral of this electric field from some point a to some other pt. b as shown in figure.

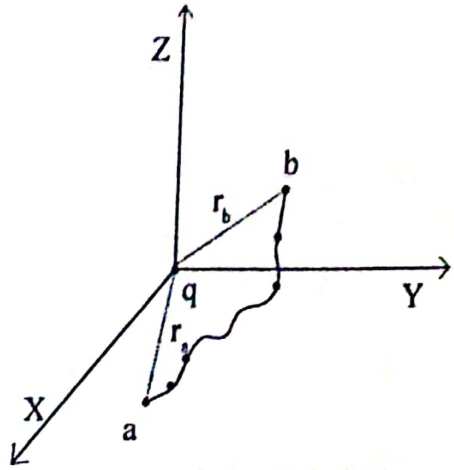
r_a = dist of pt. a from origin &

r_b = dist of pt. b from origin

In spherical co-ordinates i.e. (r, θ, ϕ)

line element $dl = dr\,\hat{r} + r d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$

$$\therefore E.dl = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}dr$$



[Fig. Curl of E]

$$\therefore \int_a^b E.dl = \frac{1}{4\pi \epsilon_0}\int_a^b \frac{q}{r^2}dr = \frac{-1}{4\pi \epsilon_0}\frac{q}{r}\Bigg]_{r_a}^{r_b} = \frac{1}{4\pi \epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right) \quad \text{---- (2)}$$

The integral around a closed path is evidently zero (for then $r_a = r_b$)

$$\oint E.dl = 0 \quad \text{---- (3)}$$

and hence applying Stoke's theorem which converts line integral to surface integral we get

$$\oint E.dl = \int_S (\nabla \times E).da = 0$$

$$\therefore \boxed{\nabla \times E = 0} \quad \text{---- (4)}$$

Moreover, if we have many charges, then total field $E = E_1 + E_2 + E_3 + ...$

So,

$$\nabla \times E = \nabla \times (E_1 + E_2 + E_3 + ...)$$

$$= (\nabla \times E_1) + (\nabla \times E_2) + (\nabla \times E_3) + ...$$

$$= 0$$

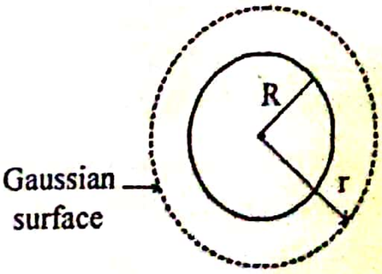
1.7 Applications of Gauss's Law :

When a symmetry permits, Gauss's law gives quickest & easiest way of computing electric fields. It is illustrates with some examples.

(i) To find electric field due to uniform charged sphere :

Consider a uniformly charged solid sphere of radius R & total charge q.

Draw a spherical surface of radius $r > R$ this is called a "Gaussian surface" For this surface According to Gauss's law.



[Fig. Electric field due to uniform charged sphere]

$$\oint_S E.da = \frac{1}{\epsilon_0} Q_{encl}$$

& $\quad Q_{encl} = q$

$$\therefore \oint_S E.da = \frac{q}{\epsilon_0}$$

$\therefore$ E point radially outwards, and da also so we can drop the dot product.

$$\therefore \oint_S E.da = \int_S |E|\, da$$

and magnitude of E is constant over the Gaussian surface.
So taking it outside the integral.

$$\int_S |E|\, da = |E| \int_S da = |E|\, 4\pi r^2$$

Thus $|E|\, 4\pi r^2 = \frac{1}{\epsilon_0} q$

or $\quad E = \frac{q}{4\pi \epsilon_0 r^2} \hat{r}$

i.e. $\quad E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \hat{r}$

The field outside the sphere is exactly the same as it would have been if all the charges had been concentrated at the centre.

ii) Application of Gauss's law, To find Electric field due to charged cylinder :

Consider a long cylinder which carries a charge density that is proportional to the from the axis; $\rho = KS$, for some constant K. The electric field inside this cylinder can be obtained using Gauss's law.

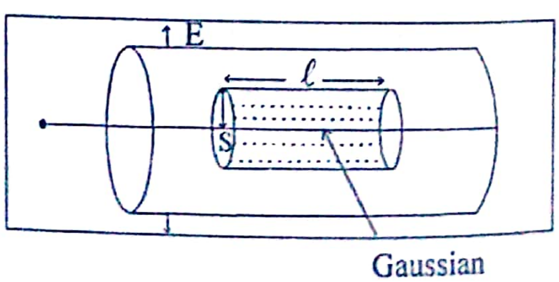
Draw Gaussian cylinder of length ℓ & radius s. For this surface Gauss's law states :

$$\int_S E.da = \frac{1}{\epsilon_0} Q_{encl}$$

The enclosed charge is

$$Q_{encl} = \int \rho d\tau = \int (KS')(S' dS' d\phi\, dZ)$$

$$= 2\pi K \ell \int_0^S S'^2 dS'$$



[Fig. Electric field to due to charged cylinder]

$$= \frac{2}{3}\pi K \ell S^3$$

(Here we used cylindrical co-ordinates viz (S, ϕ, Z) & integrated ϕ from 0 to 2π ; from dZ from 0 to ℓ. We put prime on integration variable S' to distinguish it from radius s of the Gaussian surface)

Now, because of symmetry $\overline{E}$ must point radially outwards, so for the curved portion of the Gaussian cylinder we have :

$$\int E.da = \int |E|\, da = |E| \int da = |E|\, 2\pi S \ell$$

Here E is $\perp^r$ to da & the two ends contribute nothing. Thus

$$|E| = 2\pi S\ell = \frac{1}{\epsilon_0}\frac{2}{3}\pi K \ell S^3$$

where $\frac{2}{3}\pi K \ell S^3$ is enclosed charge Q_{encl}

$$\therefore E = \frac{1}{3\epsilon_0} K S^2 \widehat{S}$$

Note :

1) The electric field of sphere falls off like $\frac{1}{r^2}$.

2) The electric field of an infinite line falls off like $\frac{1}{r}$.

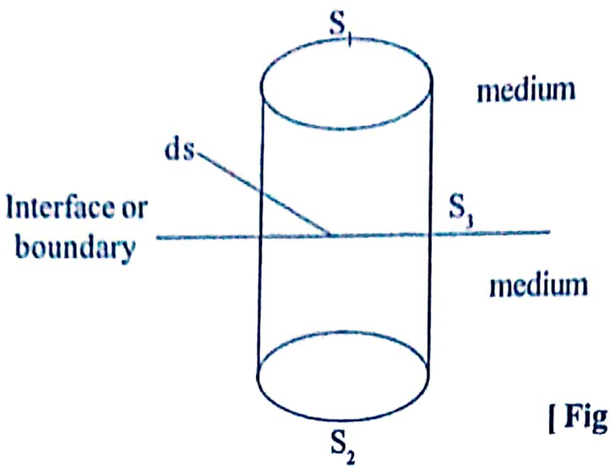
3) The electric field of an infinite plane does not fall off at all.

1.8 Gaussian Pill box :

Electric field intensity $\overline{E}$ depends on charge distribution & medium and electric displacement $\overline{D}$ is a vector independent of the properties of medium. In order to study the changes of these vectors at the boundary or interface of the media with different properties a Gaussian pill box is used.

Pill box is a cylindrical box consisting of one cylindrical surface S_3 & two plane surfaces S_1 & S_2 of circular shape on two sides of the cylindrical surface.

The area ds of the boundary surface is enclosed in the box and the two plane circular faces are in the two media on the two sides of boundary surface naturally the two plane faces will also be of area ds.

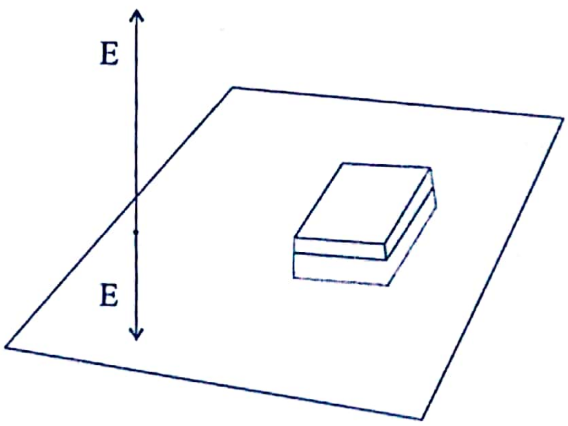


[Fig : Gaussian pill box]

The volume of the box is supposed to tend to zero by supposing that height of the box tends to zero. i.e. $h \to 0$. Therefore the contribution to the flux through the curved surface becomes zero. The pill box has contributions to flux only from its plane surfaces. The pill box is used to study the boundary conditions on field vectors $\overline{E}$ & $\overline{D}$ by applying Gauss's law to Gaussian surface.

3) If an infinite plane carries a uniform surface charge σ :

For this purpose draw a "Gaussian Pill box" entending equal distances above and below the planes as shown in figure below.



Applying Gauss's law to this surface,

$$\oint E.da = \frac{1}{\epsilon_0} Q_{encl}$$

In this case $Q_{encl} = \sigma A$ where A is the area of the lid of the pill box. By symmetry E points away from the plane thus the top & bottom surfaces yield.

$$\oint E.da = 2A\,|\,E\,|$$

where as the sides contribute nothing, thus

$$2A\,|\,E\,| = \frac{1}{\epsilon_0}\sigma A$$

$$\therefore E = \frac{\sigma}{2\epsilon_0}\hat{n}$$

where $\hat{n}$ is unit vector pointing away from the surface.

1.9 Poisson's Equation :

We have the differential form of Gauss's law

$$\nabla.E = \frac{\sigma}{\epsilon_0}\rho \qquad \text{----- (1)}$$

Furthermore, in a purely electrostatic field, $\vec{E}$ may be expressed as minus the gradient of potent.

ψ : i.e. $E = -grad\,\psi$

but $grad\,\psi = \nabla\psi$

So $E = -\nabla\psi$ ----- (2)

Combining equation (1) & (2), we obtain

$$\nabla.\nabla\psi = \frac{-\rho}{\epsilon_0} \quad \text{----- (3)}$$

Now $\nabla.\nabla = \nabla^2$ is Laplacian it is an operator. So equation (3) can be written as

$$\boxed{\nabla^2\psi = \frac{-\rho}{\epsilon_0}} \quad \text{----- (4)}$$

Laplacian operator ∇^2 is a scalar differential operator & equation (4) is a differential equation This is Poisson's equation.

∇^2 involves differentiation with respect to more than one variables; hence Poisson's equation is a partial differential equation which may be solved once we know the functional dependence of $\rho(x, y, z)$ and the appropriate boundary conditions.

In order to solve a specific problem, we must write ∇^2 in terms of x, y, z or r, θ, ϕ or etc. The choice of particular set of coordinates is arbitrary.

In Rectangular Coordinates :

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \quad \text{------ (5)}$$

In Spherical Coordinates :

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \quad \text{----- (6)}$$

and in cylindrical coordinates :

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2} \quad \text{----- (7)}$$

In spherical coordinates r is the magnitude of radius vector from origin and θ is the polar angle. In cylindrical coordinates, r is perpendicular distance from the cylinder axis & θ is azimuthal angle about the axis.

1.10 Laplace's Equations :

In a certain class of electrostatic problems involving conductors all the charges is found either on the surface of the conducts or in the form of fixed point charges. In these cases ρ is zero at most points in the space. And where the charge density vanishes. Poisson's equation reduces to the simpler form

$$\nabla^2\psi = 0 \qquad \text{------ (8)}$$

Which is Laplace's equation

If ψ is a function of one variable only, Laplace's equation reduces to an ordinary differential equation. Consider the case where ψ is $\psi(x)$ a function of the single rectangular coordinate x, then Laplace's equation is $\frac{\partial^2\psi}{\partial x^2} = 0$ and its solution is $\psi(x) = ax + b$ the equation for straight line. is the general solution where a & b are constants chosen to fit the boundary conditions.

In spherical coordinates where ψ is function of r i.e. $\psi = \psi(r)$, Laplace's equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = 0 \text{; and is } \psi(r) = \frac{-a}{r} + b$$

is a solution of this equation.

1.11 Uniquencess Theorem :

Theorem : I

If $\psi_1, \psi_2, \psi_3, ...\psi_n$ are all solutions of Laplace's equation, then

$$\psi = C_1\psi_1 + C_2\psi_2 + ...C_n\psi_n$$

Where C's are arbitrary constants, is also a solution.

The proof of this follows immediately from the fact that Laplace's equation $\nabla^2\psi = 0$ can be written as,

$$\nabla^2\psi = \nabla^2(C_1\psi_1 + C_2\psi_2 + ...C_n\psi_n) = 0$$

$$= \nabla^2 C_1\psi_1 + \nabla^2 C_2\psi_2 + ...\nabla^2 C_n\psi_n = 0$$

$$= C_1\nabla^2\psi_1 + C_2\nabla^2\psi_2 + ...C_n\nabla^2\psi_n = 0$$

$$= 0$$

Theorem : II (Uniqueness Theorem):

Two solutions of Laplace's equation that satisfy the same boundary conditions differ at most by an additive constant.

To prove this theorem we consider the closed region V_0 enterior to the surfaces $S_1, S_2, S_3, ...S_n$ of the various conductors in the problem and bounded on the outside by surface S. S being either a surface at infinity or real physical surface which encloses V_0.

Let us assume that ψ_1 and ψ_2 are two solutions of Laplace's equation in V_0 which in addition, have the same boundary conditions on $S_1, S_2, S_3 ....S_n$. These boundary conditions may be specified by assigning values of either ψ or $\partial \psi / \partial n$ on the bounding surfaces.

We define a new function $\psi = \psi_1 - \psi_2$

$$\therefore \nabla^2 \psi = \nabla^2 \psi_1 - \nabla^2 \psi_2 = 0 \text{ in } V_0$$

Furthermore, either ψ or n. $\nabla \psi$ vanishes on the boundaries. Now let us apply the divergence theorem to the vector $\psi \nabla \psi$.

$$\therefore \int_{V_0} \nabla.(\psi \nabla \psi) dv = \int_{S_1+S_2+.....S_N} \psi \nabla \psi.nda$$

$$= 0$$

Since the second integral vanishes. The divergence may be expanded by using vector identity

$$\nabla.(\psi \nabla \psi) = \psi \nabla^2 \psi + (\nabla \psi)^2$$

but $\nabla^2 \psi$ vanishes at all pts. in V_0. So divergence theorem reduces to

$$\int_{V_0} (\nabla \psi)^2 dV = 0$$

Now $(\nabla \psi)^2$ must be either positive or zero at each point in V_0 and since its integral is zero, $(\nabla \psi)^2 = 0$ is only possibility. A theorem is essentially proved.

A function whose gradient is zero at all points can not change, hence at points in V_0, ψ has the same value that it has on the bounding surfaces. If the boundary conditions have been given by specifying ψ_1 & ψ_2 on the surfaces $S, S_1, S_2 .....S_N$, then since $\psi = 0$ on these surfaces it vanishes throughout V_0. If boundary conditions are given in terms of $\partial \psi_1 / \partial n$, $\partial \psi_2 / \partial n$ then $\nabla \psi$ equals zero at all points in V_0 and $\nabla \psi.n = 0$ on the boundaries. The only solution compatible with the last statement is ψ is equal to a constant.

:: Multiple Choice Questions ::

1. If the electric field is directed outward at every point on the closed surface, the total electric flux will be positive and form Gauss's law, there ....
   a) **must be net positive charge within the surface.**
   b) must be net negative charge within the surface.
   c) should be no charge on the surface.
   d) must be both positive as well as negative charge with in the surface.

2. Poisson's equation is ....
   a) $\nabla \cdot E = \frac{\rho}{\epsilon_0}$
   b) $\nabla^2 \psi = 0$
   c) $E = -\nabla \psi$
   d) **none of these.**

3. Sol^n to Laplace's eq^n in one independent variable in rectangular cooedinates is ......
   a) $\psi(r) = -\frac{a}{r} + b$
   b) **$\psi(x) = ax + b$**
   c) $\psi(r) = -\frac{b}{r}$
   d) none of these.

4. In a purly electrostatic field, E may be expressed as ....
   a) minius the gradient of potential ψ
   b) minius the of potential ψ
   c) $-\frac{\rho}{\epsilon_0}$
   d) none of these.

5. The lines of force for single +ve charge q are ....
   a) radial lines radiating outwards from charge q.
   b) radial lines radiating inwards from charge q.
   c) **along a straight line.**
   d) All of the above.

6. $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ ......
   a) Divergenence theorem
   b) Stoke's theorem.
   c) **Gauss's law in differential form.**
   d) Gauss's law in integral form.

7. The electric field produced by a distribution of charge.

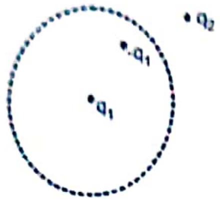
a) **Michael Faraday**

b) Gauss.

c) Coulomb

d) None of these.

8. The Gaussian surface in the fig. is shown by dotted line. The electric field on the surface will be ...



a) due to q_1 & q_2

b) due to q_2 only

c) zero

d) due to all.

9. The electric force experiented by a unit positive charge placed at that point is called as the intensity of ....

a) electric field.

b) Magnetic field.

c) Electromagnetic field.

d) None of these.

10. According to Gauss law,the flux through any surface enclosing the charge is .....

a) $q.\epsilon_0$

b) $\frac{q}{\epsilon_0}$

c) $q_0 \epsilon_0$

d) None of these.

12. The electric field outside a uniformly charged solid sphere of radius R is ....

a) $\frac{\rho R^3}{3\epsilon_0 r^2}$

b) $\frac{\rho R^2}{3\epsilon_0 r^2}$

c) $\frac{\rho R^3}{3\epsilon_0 r^3}$

d) None of these.

13. The electric field for a point on the uniformly charged solid sphere of radius R is ....

a) $\frac{\rho}{4\pi R^2 \epsilon_0}$

b) $\frac{q}{4\pi R^2 \epsilon_0}$

c) $\frac{q^2}{4\pi R^3 \epsilon_0}$

d) None of these.

14. The electric field for a point inside the uniformly charged solid sphere of radius R is ....

a) $\frac{\rho r}{3 \epsilon_0}$ b) $\frac{\rho}{3 \epsilon_0}$

c) $\frac{\rho r^2}{3 \epsilon_0}$ d) None of these.

15. In electric field due to a charge cylinder, if e > R then electric field is ....

a) $\frac{n\lambda}{2\pi r \epsilon_0}$ **b)** $\frac{\lambda}{2\pi r \epsilon_0}$

c) $\frac{\lambda}{2\pi r^2 \epsilon_0}$ d) None of these.

16. "Gaussian pillbox" extending equal distance .....

a) above the plane. b) below the plane.

c) above and below the plane. d) None of these.

:: Questions ::

1. Explain the concept of electric field lines. Gives its properties.
2. Explain the concept of electric flux or flux of electric field.
3. State and prove Gauss's law in electrostics.
4. Derive an expression for divergence of E & curl of E.
5. By applying Gauss law. Find electric field due to uniform charged sphere.
6. Derive an expression for electric field due to charged cylinder.
7. Explain the concept of Gaussian pill box.
8. Show that poissons eq^n $\nabla^2 t = {}^{-\rho}/_{\epsilon_0}$ for cylindrical spherical & rectangular co-ordinate system.
9. Give the concept of Laplaces eq^n.
10. State and explain the uniqueness theorem.

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